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**Corrections in Initial Printing**

**Note to Instructors:** For brevity and compactness, only the most essential steps are included in these problem solutions. This manual was typeset in EXACT™, and computer solutions were run using MathCAD™. Print-outs of the computer solutions are included in this manual at a reduced size. A diskette with solutions to computer problems, for use with the student version of MathCAD, is available from the publisher. An optional list of answers to selected problems in the text is available from the author.

Please address comments and corrections concerning this manual to:
Prof. Ferrel G. Stremler, Dept. of Electrical and Computer Engineering,
University of Wisconsin, 1415 Johnson Drive, Madison, WI 53706-1691.
CHAPTER 2
ORTHOGONALITY AND
SIGNAL REPRESENTATIONS

DP2.4.1  a) \( \mathbf{A} \cdot \mathbf{D} = 1 - 5 - 20 = -24 \)
\( \mathbf{B} \cdot \mathbf{D} = -1 + 5 - 4 = 0 \)
\( \mathbf{C} \cdot \mathbf{D} = 3 + 5 - 8 = 0 \)
Therefore \( \mathbf{B} \) and \( \mathbf{C} \) are orthogonal to \( \mathbf{D} \).

b) Note that \( \mathbf{B} \cdot \mathbf{C} = 0 \), \( \mathbf{B} \cdot \mathbf{D} = 0 \), \( \mathbf{C} \cdot \mathbf{D} = 0 \)

Let: \( A_1 = \frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{B} \cdot \mathbf{B}} = 1; \) \( A_2 = \frac{\mathbf{A} \cdot \mathbf{C}}{\mathbf{C} \cdot \mathbf{C}} = \frac{6}{7}; \) \( A_3 = \frac{\mathbf{A} \cdot \mathbf{D}}{\mathbf{D} \cdot \mathbf{D}} = \frac{4}{7}; \)

so that: \( \mathbf{A} = A_1 \mathbf{B} + A_2 \mathbf{C} + A_3 \mathbf{D} = \mathbf{B} + \frac{6}{7} \mathbf{C} - \frac{4}{7} \mathbf{D}. \)

c) \( \mid \epsilon \mid^2 = \mid \mathbf{A} - A_2 \mathbf{C} - A_3 \mathbf{D} \mid^2 = \mid -x_1 + x_2 + x_3 \mid^2 = 3. \)

DP2.5.1 Using Eqs. (2.22), (2.26) for this case gives: \( f(t) = \sum_{n=0}^{\infty} f_n \cos \frac{n\pi}{4} t, \)

\[ f_n = \frac{\int_{-2}^{2} \cos(n\pi/4) t \, dt}{\int_{-4}^{4} \cos^2(n\pi/4) t \, dt} = \begin{cases} 1/2 & n=0 \\ \frac{\sin(n\pi/2)}{(n\pi/2)} & n \neq 0 \end{cases} \]

DP2.6.1 Substituting into the differential equation and performing the derivative operation, we have:
\( sV_{o e s t} + \frac{R}{L} V_{o e s t} = \frac{R}{L} V_{i e s t} \quad \text{or}, \quad V_{o}/V_{i} = (R/L)/(s + R/L). \)

DP2.7.1 Using Eq. (2.34), \( \omega_0 = 2\pi/8 = \pi/4 \), and using Eq. (2.36),

\[ F_n = \frac{1}{8} \int_{-2}^{2} \exp(-jn\pi/4) t \, dt = \frac{1}{2} \frac{\sin(n\pi/2)}{(n\pi/2)} \]

The average value of the waveform over (-4, 4) is 1/2 (i.e., \( F_0 = 1/2 \)).

DP2.8.1 Let: \( f_1 = f_{r_1} + jf_{i_1}; \) \( f_2 = f_{r_2} + jf_{i_2}; \)

then: \( f_1 f_2 = f_{r_1} f_{r_2} - f_{i_1} f_{i_2} + jf_{r_1} f_{i_2} + jf_{i_1} f_{r_2} \)
and: \( \Re(f_1 f_2) = f_{r_1} f_{r_2} - f_{i_1} f_{i_2} = \Re(f_1)\Re(f_2) - \Im(f_1)\Im(f_2). \)

**DP2.9.1** Aside from a dc level, \( f(t) = \pi(1 - |t|/\pi) \) over \((-\pi, \pi)\) and \( \omega_0 = 2\pi/(2\pi) = 1. \) Because \( f(t) \) is even, its trigonometric Fourier series has cosine terms only. Using Eq.(2.54) and properties of even functions,

\[
a_n = \frac{4}{2\pi} \int_0^\pi (1 - t/\pi) \cos nt \, dt = \left\{ \begin{array}{ll} 4/(n\pi^2) & \text{for odd } n, \\ 0 & \text{for even } n. \end{array} \right.
\]

The dc level of the waveform \( f(t) \) is zero by inspection.

**DP2.9.2** By subtracting out the dc level, note that \( f(t) \) is an odd function of \( t \) and therefore its trigonometric Fourier series has sine terms only; also, \( \omega_0 = 2\pi/2 = \pi; \)

\[
b_n = \frac{4}{2\pi} \int_0^\pi \sin nt \, dt = \frac{1}{n\pi^2} \left[ \sin n\pi t - n\pi t \cos n\pi t \right]_0^\pi = -\frac{2}{n\pi^2}.
\]

The dc level of the waveform is one (1) by inspection (i.e., \( a_0 = 1 \)).

**DP2.10.1** The waveform \( f(t) \) is an even function of \( t \) so there are cosine terms only in its trigonometric Fourier series representation. Shifting the waveform upwards by one unit for convenience in computation, we get:

\[
a_n = \frac{4}{T} \int_0^{T/4} 2 \cos n\omega_0 T/4 \, dt = 2 \left. \frac{\sin n\omega_0 T/4}{n\omega_0 T/4} \right|_0^{T/4} = 2 \frac{\sin \pi/2}{n\pi/2}, \quad n\neq 0.
\]

The dc level of the waveform is zero by inspection (i.e., \( a_0 = 0 \)).

**DP2.11.1** \( f(t) = 2 \cos 100t = \exp(j100t) + \exp(-j100t) \), from which \( F_1 = F_{-1} = 1 \) and \( F_n = 0 \) for \( n \neq \pm 1 \). Using Eq.(2.68), \( P = |F_{-1}|^2 + |F_1|^2 = 1 + 1 = 2 \). There is a spectral line (=1) at \( \omega = -100 \) rad/sec and a spectral line (=1) at \( \omega = +100 \) rad/sec.

**DP2.12.1** Let: \( f(t) = \exp(j\omega t) \), and \( g(t) = H(\omega) \exp(j\omega t) \); then:\n
\[
H(\omega) \exp(j\omega t) + (j\omega)^4 H(\omega) \exp(j\omega t) = (j\omega)^2 \exp(j\omega t),
\]

or, cancelling common factors, \( H(\omega) = -\omega^2/(1 + \omega^4) \).

**DP2.13.1** a) Using Eq.(2.77), \( g(t) = \sum_{n=-\infty}^{\infty} \frac{-2\pi n^2}{1 + (2\pi n)^4} \frac{\sin \pi n/2}{\pi n/2} e^{j2\pi nt} \), or,\n
\[
g(t) \approx \sum_{n=-\infty}^{\infty} \frac{-\sin \pi n/2}{2(n\pi)^3} e^{j2\pi nt} = \frac{1}{\pi^3} \left( -\cos 2\pi t + \frac{1}{3} \cos 6\pi t - \frac{1}{5} \cos 10\pi t + \ldots \right)
\]

b) Using Eq.(2.78), \( P = 2 \sum_{n=1}^{\infty} \left[ \frac{(2\pi n^2)^2}{1 + (2\pi n)^4} \right] \frac{2}{n^2} \approx 0.52 \text{ mW} \).

**DP2.14.1** a) From Example 2.5.1, the first term in the trigonometric Fourier series for the output waveform is: \((4/\pi) \sin \omega_0 t\). Because the
input is: \((2 \times 10^{-3}) \sin \omega_0 t\), the linear gain is: \((4/\pi)/(2 \times 10^{-3}) = 637\).

b) The mean-square value of the output waveform is: 
\[
\overline{f^2} = \frac{1}{T} \int_{-T/2}^{T/2} f^2(t) \, dt = 1
\]
so that: 
\[
\text{THD} = \frac{\overline{f^2} - b_1^2/2}{b_1^2/2} = \frac{1 - (4/\pi)^2/2}{(4/\pi)^2/2} = 0.234.
\]

**DP2.16.1** Computer problem.

**DP2.17.1** Computer problem.

**DP2.17.2** a) The spectrum of the sampled waveform repeats at multiples of \(M\) so that the coefficients for \(n = M, 2M, \ldots\), are the same as those at \(n = 0\).
b) Because there is complex conjugate (i.e., Hermitian) symmetry about \(n = 0, M/2, M, 3M/2, \ldots\), the complex conjugate at those points is equal to itself; i.e., the coefficients at those points must be real-valued.

**DP2.18.1** a) Because the interval of integration does not include \(\delta(x)\), we have: 
\[
\int_{1}^{100} e^{-x^2} \delta(x) \, dx = \int_{1}^{1} e^{-x^2} (0) \, dx = 0.
\]
b) 
\[
\int_{1}^{1} \log t \delta(t - 10) \, dt = \log (10) = 1.
\]

**DP2.18.2** Using Eqs. (2.98), (2.99), 
\[
\int_{-\infty}^{\infty} \frac{1}{\pi} \delta(t - \frac{1}{\pi}) \cos\left(\frac{1}{\pi} t\right) \, dt = \frac{1}{\pi} \cos \pi = -\frac{1}{\pi}.
\]

**DP2.18.3** 
\[
\int_{-\infty}^{\infty} [u(t) - u(t - 3)] \, dt - K \int_{-\infty}^{\infty} \delta(t - 4) \, dt = \int_{0}^{3} \, dt = K = 0, \text{ or } K = 3.
\]

**DP2.19.1** a) Using: \(f(t) = \delta(t)\), we have: \(g(t) = \delta(t) + \delta(t-T) + \ldots\)
By definition, \(g(t) = h(t)\) for \(f(t) = \delta(t)\) so that: 
\[
h(t) = \sum_{n=0}^{\infty} \delta(t - nT).
\]
b) Using: \(f(t) = \delta(t)\), we have: 
\[
g(t) = \delta(t) + K\delta(t-T) + K^2\delta(t-2T) + \ldots
\]
or, \(h(t) = \sum_{n=0}^{\infty} K^n \delta(t - nT)\).

**P2.2.1** a) Power; 
\[
P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} (4)^2 \, dt = 16 \text{ W}.
\]
b) Power; 
\[
P = \frac{1}{T} \int_{-T/2}^{T/2} (\cos t + \cos 2t)^2 \, dt = 0.5 + 0.0 + 0.5 = 1 \text{ W}.
\]
c) Energy; 
\[
E = 2 \int_{0}^{\infty} e^{-4t} \, dt = \frac{-1}{2} e^{-4t} \bigg|_{0}^{\infty} = 0.5 \text{ J}.
\]
d) Power; 
\[
P = \frac{1}{T} \int_{-T/2}^{T/2} e^{j2\pi t} e^{-j2\pi t} \, dt = \frac{1}{T} \int_{-T/2}^{T/2} \, dt = 1 \text{ W}.
\]
P2.2.2  a) Periodic; \( \sqrt{3} T = 2\pi \), or \( T = 2\pi/\sqrt{3} \approx 3.63 \) sec.
b) Periodic; \( 2\pi T = 2\pi \), or \( T = 1 \) sec.
c) Periodic; \( \sin^2(2t) = (1/2) - (1/2)\cos(4t); 4T = 2\pi, T = \pi/2 \approx 1.57 \) sec.
d) Nonperiodic; \( f(t) \neq f(t+T) \).

P2.3.1  a) linear, time-invariant, causal.
b) linear, time-invariant, noncausal.
c) linear, time-varying, causal.
d) nonlinear, time-invariant, causal.

P2.4.1  a) \( |\vec{\phi}_1| = \sqrt{2} \neq 0; \ |\vec{\phi}_2| = \sqrt{2} \neq 0; \ |\vec{\phi}_3| = 1 \neq 0. \)
\[ \vec{\phi}_1 \cdot \vec{\phi}_2 = 1 - 1 = 0; \ \vec{\phi}_1 \cdot \vec{\phi}_3 = 0; \ \vec{\phi}_2 \cdot \vec{\phi}_3 = 0; \] Eq.(2.17) is satisfied.
b) We can write: \( \vec{C} = C_{11}\vec{\phi}_1 + C_{12}\vec{\phi}_2 + C_{13}\vec{\phi}_3 \) \[ \text{[Eq.(2.18)]} \]
where: \( C_{11} = \frac{\vec{C} \cdot \vec{\phi}_1}{\vec{\phi}_1 \cdot \vec{\phi}_1} = 2; \ \ C_{12} = \frac{\vec{C} \cdot \vec{\phi}_2}{\vec{\phi}_2 \cdot \vec{\phi}_2} = 1; \ \ C_{13} = \frac{\vec{C} \cdot \vec{\phi}_3}{\vec{\phi}_3 \cdot \vec{\phi}_3} = 2; \)
so: \( \vec{C} = 2\vec{\phi}_1 + \vec{\phi}_2 + 2\vec{\phi}_3. \) Similarly, \( \vec{D} = 3\vec{\phi}_1 - 2\vec{\phi}_2 - 4\vec{\phi}_3. \)

P2.4.2  a) \( \vec{Y} \cdot \vec{\phi}_3 = c = 0; \ \vec{Y} \cdot \vec{\phi}_2 = a - b, \ or \ a = b. \)
b) \( \vec{Y} \cdot \vec{\phi}_1 = a + b = 0, \ or \ a = -b. \)
c) Using results of (a), plus \( |\vec{Y}| = \sqrt{2}, a = b, c = 0, \) and \( a = 1. \)

P2.5.1  a) \( f(t) = \sum_n f_n \phi_n(t), \) so that
\[
\int_{t_1}^{t_2} \epsilon_1(t)\phi_1^*(t) \, dt = \int_{t_1}^{t_2} [\sum_n f_n \phi_n(t) - f_1 \phi_1(t)]\phi_1^*(t) \, dt.
\]
Interchanging order, and noting that \( \phi_n(t) \) are orthogonal, we have:
\[
\int_{t_1}^{t_2} \epsilon_1(t)\phi_1^*(t) \, dt = \sum_n f_n \int_{t_1}^{t_2} \phi_n(t)\phi_1^*(t) \, dt - f_1 \int_{t_1}^{t_2} \phi_1(t)\phi_1^*(t) \, dt = 0.
\]
b) \( \int_{t_1}^{t_2} |\epsilon_1(t)|^2 \, dt = \int_{t_1}^{t_2} [\sum_n f_n \phi_n(t) - f_1 \phi_1(t)]^2 \, dt, \) or,
\[
\int_{t_1}^{t_2} |\epsilon_1(t)|^2 \, dt = \sum_n f_n \sum_m f_m^* \int_{t_1}^{t_2} \phi_n(t)\phi_m^*(t) \, dt - f_1^* f_1 \sum_n \int_{t_1}^{t_2} \phi_n(t)\phi_1^*(t) \, dt.
\]
\[ -f_1 \sum_{m} f_m^* \int_{t_1}^{t_2} \phi_m(t) \phi_m^* dt + f_1 f_1^* \int_{t_1}^{t_2} \phi_1(t) \phi_1^*(t) dt. \]

As a result of orthogonality, this can be rewritten as:
\[ \int_{t_1}^{t_2} |\epsilon_1(t)|^2 dt = \sum_{n} |f_n|^2 \int_{t_1}^{t_2} |\phi_n(t)|^2 dt - |f_1|^2 \int_{t_1}^{t_2} |\phi_1(t)|^2 dt. \]

Interchanging and regrouping yields:
\[ \int_{t_1}^{t_2} |\epsilon_1(t)|^2 dt = \int_{t_1}^{t_2} |f(t)|^2 dt - \int_{t_1}^{t_2} |f_1 \phi_1(t)|^2 dt. \]

**Problem 2.5.2**
\[ f(t) = f_1 \phi_1(t) + f_2 \phi_2(t) + f_3 \phi_3(t), \] where:
\[ f_1 = \frac{\int_{0}^{1} \sin 2\pi t \phi_1(t) dt}{\int_{0}^{1} |\phi_1(t)|^2 dt}; \quad f_2 = \frac{\int_{0}^{1} \sin 2\pi t \phi_2(t) dt}{\int_{0}^{1} |\phi_2(t)|^2 dt}; \quad f_3 = \frac{\int_{0}^{1} \sin 2\pi t \phi_3(t) dt}{\int_{0}^{1} |\phi_3(t)|^2 dt}. \]

**Problem 2.5.3**
\[ a) \int_{0}^{1} \phi_0(t) \phi_1(t) dt = 0; \quad \int_{0}^{1} \phi_0(t) \phi_2(t) dt = 0; \quad \int_{0}^{1} \phi_1(t) \phi_2(t) dt = 0; \]
\[ \text{and} \quad \int_{0}^{1} \phi_0^2(t) dt = \int_{0}^{1} \phi_1^2(t) dt = \int_{0}^{1} \phi_2^2(t) dt = 1. \]

Thus this set is not only orthogonal; it is orthonormal over (0,1).

b) \[ f(t) \approx \sum_{n=0}^{2} f_n \phi_n(t) \] over (0,1), where:
\[ f_n = \int_{0}^{1} f(t) \phi_n(t) dt. \]

Evaluating, we get: \[ f(t) \approx \phi_0(t) - \frac{1}{2} \phi_1(t) - \frac{1}{4} \phi_2(t) \] over (0,1).

d) \[ E_0 = (1)^2 (1) = 1; \]
\[ E_1 = (1/2)^2 (1) = 1/4; \]
\[ E_2 = (1/4)^2 (1) = 1/16. \]

The energy in the error remaining after each successive term is
[see Prob.2.5.1(b)]:

Original energy = \[ \int_{0}^{1} (2t)^2 dt = 4/3; \] after \( n = 0 \) term: \( 4/3 - 1 = 1/3; \)
after \( n = 1 \): \( 1/3 - 1/4 = 1/12; \) after \( n = 2 \): \( 1/12 - 1/16 = 1/48. \)

**Problem 2.5.4**
\[ a) \int_{-1}^{1} \phi_0(t) \phi_1(t) dt = 0; \quad \int_{-1}^{1} \phi_0(t) \phi_2(t) dt = 0; \quad \int_{-1}^{1} \phi_1(t) \phi_2(t) dt = 0; \]
\[ \int_{-1}^{1} \phi_0^2(t) \, dt = 2; \quad \int_{-1}^{1} \phi_1^2(t) \, dt = 2/3; \quad \int_{-1}^{1} \phi_2^2(t) \, dt = 2/5. \]

b) \( f(t) \approx \sum_{n=0}^{2} f_n \phi_n(t) \) over \((-1,1)\), where: \( f_0 = \int_{-1}^{1} t \, dt / \int_{-1}^{1} dt = 1/4, \)
\[
f_1 = \int_{-1}^{1} t^2 \, dt / \int_{-1}^{1} t^2 \, dt = 1/4, \quad f_2 = \int_{0}^{1} (3t^3/2 - t/2) \, dt / \int_{-1}^{1} \phi_2^2(t) \, dt = 5/16.
\]
\[
f(t) \approx \left(\frac{1}{4}\right) \phi_0(t) + \left(\frac{1}{2}\right) \phi_1(t) + \left(\frac{5}{16}\right) \phi_2(t).
\]

c) The two graphs are shown.

\[ \text{P2.7.1} \]
a) \( n \omega_0 = n \pi \), from which: \( T = 2\pi/\omega_0 = 2 \) seconds.

b) \( \text{avg.} = F_0 = 1 - e^{-a} \) (by inspection). We can also write:
\[
\text{avg.} = \frac{1}{T} \int_{-T/2}^{T/2} e^{-a|t|} \, dt = \int_{0}^{1} e^{-at} \, dt = 1 - e^{-a}.
\]

\[ c) \quad \frac{2a^2(1 - e^{-a} \cos 3\pi)}{a^2 + 9\pi^2} \cdot \frac{e^{j3\pi t}}{2} + \frac{2a^2(1 - e^{-a} \cos 3\pi)}{a^2 + 9\pi^2} \cdot \frac{e^{-j3\pi t}}{2} = A \cos 3\pi t, \quad \text{or} \quad A = \frac{2a^2(1 - e^{-a} \cos 3\pi)}{a^2 + 9\pi^2}. \]

d) \( f(t) = (1 - e^{-a}) + \sum_{n=1}^{2} A \cos n\pi t; \quad f(0) = (1 - e^{-a}) + 2a^2 \left[ \frac{1 + e^{-a}}{a^2 + \pi^2} + \frac{1 - e^{-a}}{a^2 + 4\pi^2} \right]. \]

\[ \text{P2.7.2} \]
f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_0 t}, \text{ where: } F_n = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} A \cos \omega_0 t e^{-j\omega_0 t} \, dt; \text{ as a result of orthogonality, } F_n = \frac{A}{2(t_2 - t_1)} \int_{t_1}^{t_2} (e^{j\omega_0 t} + e^{-j\omega_0 t}) e^{-j\omega_0 t} \, dt = 0, \quad n \neq \pm 1.

For \( n = +1 \):
\[
F_1 = \frac{A}{2(t_2 - t_1)} \int_{t_1}^{t_2} (1 + e^{-j2\omega_0 t}) \, dt = \frac{A}{2}.
\]

For \( n = -1 \):
\[
F_{-1} = \frac{A}{2(t_2 - t_1)} \int_{t_1}^{t_2} (e^{j2\omega_0 t} - 1) \, dt = \frac{A}{2}.
\]
These results agree with Euler's identity [see Eq. (2.43)].

\[ \text{P2.7.3} \]
\( t_2 - t_1 = 2; \quad \omega_0 = 2\pi/2 = \pi; \quad f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\pi t}, \)
where: \( F_n = \frac{1}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 + \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2}) e^{-jn\pi t} \, dt, \)
\[ F_n = \frac{1}{2} \sin(\pi n/2) + \frac{1}{4} \sin((n-2)\pi/2) + \frac{1}{4} \sin((n+2)\pi/2). \]

P2.7.4 a) \( G_n = \frac{1}{[(t_2-t_1)/a]} \int_{t_1/a}^{t_2/a} f(\alpha t) e^{-jn(\omega_0)t} \, d\alpha. \)
Changing variable, let \( x = at \) and \( dx = a \, dt; \) then
\[ G_n = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} f(x) e^{-jn\omega_0 x} \, dx = F_n. \]
Thus, the Fourier series coefficients are invariant with time scaling.

b) \[ G_n = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} f(t - t_0) e^{-jn\omega_0 t} \, dt. \]
Changing variable, let \( x = t - t_0 \) and \( dx = dt \) so that:
\[ G_n = \frac{1}{(t_2-t_1)} \int_{t_1-t_0}^{t_2-t_0} f(x) e^{-jn\omega_0(x+t_0)} \, dx = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} f(x) e^{-jn\omega_0 x} \, dx = e^{-jn\omega_0 t_0} F_n. \]

c) \[ G_n = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} e^{jn\omega_0 t} f(t) e^{-jn\omega_0 t} \, dt = \frac{1}{(t_2-t_1)} \int_{t_1}^{t_2} e^{-j(n-1)\omega_0 t} \, dt = F_{n-1}. \]

P2.8.1 a) \( z + z^* = x + jy + x - jy = 2x = 2\Re(z). \)
b) \( z - z^* = x + jy - x + jy = 2jy = j2\Im(z). \)
c) \( (z_1 \pm z_2)^* = (x_1 + jy_1 \pm x_2 \mp jy_2)^* = x_1 - jy_1 \mp x_2 \mp jy_2 = z_1^* \mp z_2. \)
d) \( (z_1 z_2)^* = [(x_1 + jy_1)(x_2 + jy_2)]^* = [x_1 x_2 + jx_1 y_2 + jy_1 x_2 - y_1 y_2]^* = x_1 x_2 - jx_1 y_2 - jy_1 x_2 - y_1 y_2 = (x_1 - jy_1)(x_2 - jy_2) = z_1^* z_2. \)
e) \( \Re(z_1 z_2) = \Re((x_1 + jy_1)(x_2 + jy_2)) = \Re(x_1 x_2 - y_1 y_2 + jx_1 y_2 + jy_1 x_2) = x_1 x_2 - y_1 y_2 = \Re(z_1) \Re(z_2) - \Im(y_1) \Im(y_2). \)
f) \( \Im(z_1 z_2) = \Im(x_1 x_2 - y_1 y_2 + jx_1 y_2 + jy_1 x_2) = x_1 y_2 + y_1 x_2 = \Re(z_1) \Im(z_2) + \Im(z_1) \Re(z_2). \)

P2.8.2 a) Because \( f(t) \) has even symmetry, we let \( t_1 = 0 \) and use the
symmetric interval \((-t_2,t_2)\) for the integration; then

\[
F_n = \frac{1}{t_2-t_1} \int_{t_1}^{t_2} f(t) e^{-j\omega_0 t} \, dt = \frac{1}{2t_2} \int_{-t_2}^{t_2} f(t) \cos \omega_0 t \, dt - \frac{j}{2t_2} \int_{-t_2}^{t_2} f(t) \sin \omega_0 t \, dt.
\]

The second integration is that of an odd function over the symmetric interval \((-t_2,t_2)\) and gives zero, yielding the desired result.

b) Using the result of part a) above and noting that \(\omega_0 = 2\pi/T\), we get:

\[
F_n = \frac{1}{T} \int_{-T/4}^{T/4} f(t) \cos \omega_0 t \, dt = \frac{2}{T} \int_{0}^{T/4} f(t) \cos \omega_0 t \, dt,
\]

\[
F_n = \frac{2}{T} \int_{0}^{T/4} 2A \cos \omega_0 t \, dt = \frac{A}{\pi^2 \omega_0^2} \left[ \cos(n\pi/2) + (n\pi/2) \sin(n\pi/2) - 1 \right].
\]

c) Using the same approach as in part a) above, we can write:

\[
F_n = \frac{1}{2t_2} \int_{t_2}^{t_2} f(t) \cos \omega_0 t \, dt - \frac{j}{2t_2} \int_{-t_2}^{-t_2} f(t) \sin \omega_0 t \, dt.
\]

Because the product of two odd functions results in an even function, only the latter integration is nonzero over \((-t_2,t_2)\).

P2.8.3 Referring to the phasor diagram shown, we can write:

\[
\phi = \frac{1}{2} e^{j(\omega_0 t-\psi)} + \frac{1}{2} e^{-j\omega_0 t}
\]

\[
\phi = e^{-j\psi/2} \left[ \frac{1}{2} e^{j(\omega_0 t-\psi/2)} + \frac{1}{2} e^{-j(\omega_0 t-\psi/2)} \right]
\]

\[
\phi = e^{-j\psi/2} \cos(\omega_0 t - \psi/2).
\]

Therefore the lag angle is: \(\phi = \psi/2\).

P2.9.1 a) \(\omega_0 = 2\pi/2 = \pi\); \(a_0 = \frac{1}{2} \int_{-1/2}^{1/2} g \, dt = 1\);

\[
a_n = \frac{2}{2} \int_{-1/2}^{1/2} \sin \pi t \, dt = 4 \int_{0}^{1/2} \cos \pi t \, dt = 2 \frac{\sin(\pi/2)}{(\pi/2)};
\]

\[
b_n = \frac{2}{2} \int_{-1/2}^{1/2} \sin \pi t \, dt - 0 \quad \text{(integral of odd fn. over symmetric interval)}.
\]

Thus, the series is: \(f(t) = 1 + 2 \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{(n\pi/2)} \cos n\pi t\).

b) Expanding, we have: \(f(t) = 1 + \frac{4}{\pi} \cos \pi t - \frac{4}{3\pi} \cos 3\pi t + \frac{4}{5\pi} \cos 5\pi t - \cdots\).

This result has the same harmonics (i.e., multiples of \(\omega_0\)) as Example 2.5.1 but:

1) Has an average value of 1, in contrast to 0 in Example 2.5.1.
2) Expands in cosines rather than sines as a result of the even symmetry, in contrast to the odd (folded) symmetry in Example 2.5.1.

Note from a comparison with Drill Problem 2.5.1 that time scaling affects \(\omega_0\), but does not affect the Fourier series coefficients.
P2.9.2 a) \( \omega_0 = 2\pi/4 = \pi/2 \). Breaking \( f(t) \) into even and odd functions,

\[
F_n = \frac{1}{4} \int_{-1}^{1} \frac{3}{2} e^{-j(\pi n/2) t} dt + \frac{1}{4} \int_{0}^{1} \frac{1}{2} e^{-j(\pi n/2) t} dt - \frac{1}{4} \int_{0}^{1} \frac{1}{2} e^{-j(\pi n/2) t} dt,
\]
\[
F_n = \frac{3}{4} \sin(n\pi/2) + \frac{1}{8} \sin(n\pi/4) e^{j\pi n/4} - \frac{1}{8} \sin(n\pi/4) e^{-j\pi n/4},
\]
\[
F_n = \frac{3}{4} \sin(n\pi/2) + \frac{j}{4} \sin^2(n\pi/4).
\]

b) \( a_0 = \frac{1}{4} \int_{-1}^{1} 2 dt + \frac{1}{4} \int_{0}^{1} 1 dt = \frac{3}{4} \);

\[
a_n = \frac{2}{4} \int_{-1}^{1} \frac{3}{2} \cos(n\pi/2) t dt = \frac{3}{2} \int_{0}^{1} \cos(n\pi/2) t dt = \frac{3}{2} \sin(n\pi/2);
\]

\[
b_n = -\int_{0}^{1} \frac{1}{2} \sin(n\pi/2) t dt = \frac{1}{2} \frac{\cos(n\pi/2) - 1}{(n\pi/2)}.
\]

c) Comparing results of a), b) above, it is seen that [Eqs. (2.49-51)]:

\[
a_0 = F_0; \quad a_n = 2Re(F_n); \quad b_n = -2Im(F_n).
\]

---

P2.10.1 b) for \( f(t) \): \( a_0 = 0 \),

by inspection.

for \( g(t) \): \( a_0 = \frac{2}{T/2} \int_{0}^{T/4} \cos \frac{2\pi}{T} t dt \)

or, \( a_0 = \frac{\sin(\pi/2)}{(\pi/2)} = \frac{2}{\pi} = 0.637 \).

c) As a result of even symmetry,

\[
b_n = 0; \quad \text{the} \ a_n \ 's \ 's \ are:
\]

\[
@\omega_0: \quad a_1 = \frac{4}{T} \int_{0}^{T/4} \cos \omega_0 t \cos \omega_0 t dt - \frac{4}{T} \int_{T/4}^{T/2} \cos \omega_0 t \cos \omega_0 t dt = 0.
\]

\[
@2\omega_0: \quad a_2 = \frac{4}{T} \int_{0}^{T/4} \cos \omega_0 t \cos 2\omega_0 t dt - \frac{4}{T} \int_{T/4}^{T/2} \cos \omega_0 t \cos 2\omega_0 t dt
\]

\[
= \frac{4}{T} \frac{1}{3\omega_0} + \frac{1}{3\omega_0} = \frac{4}{3\pi} \quad \text{(These results agree with Table 2.2.)}
\]

---

P2.10.2 b) for \( f(t) \): \( a_0 = 0 \),

by inspection.

for \( g(t) \): \( a_0 = \frac{2}{T} \int_{0}^{T/4} \cos \frac{2\pi}{T} t dt \)

or, \( a_0 = \frac{\sin(\pi/2)}{2(\pi/2)} = \frac{1}{\pi} = 0.318 \).

c) As a result of even symmetry,

\[
b_n = 0; \quad \text{the} \ a_n \ 's \ 's \ are:
\]

\[
@\omega_0: \quad a_1 = \frac{4}{T} \int_{0}^{T/4} \cos \omega_0 t \cos \omega_0 t dt = \frac{1}{2}.
\]
@2\omega_0: \quad a_2 = \frac{4}{T} \int_0^{T/4} \cos \omega_0 t \cos 2\omega_0 t \, dt = \frac{4}{T} \left[ \frac{1}{3\omega_0} \right] = \frac{2}{3\pi}.

P2.10.3 We assume a peak amplitude of 1 V for convenience. Also assuming even symmetry (i.e., cosine terms only), we have:

\[ \overline{\varepsilon^2} = \frac{1}{T} \int_{-T/2}^{T/2} \varepsilon^2(t) \, dt = \frac{1}{T} \int_{-T/2}^{T/2} [f(t) \sum_{n=1}^N a_n \cos n\omega_0 t]^2 \, dt \]

\[ = \frac{1}{T} \int_{-T/2}^{T/2} f^2(t) \, dt - \frac{1}{2} \sum_{n=1}^N a_n^2 = 1 - \frac{1}{2} \sum_{n=1}^N a_n^2. \]

Using the result listed in Table 2.2, \( a_n = \frac{2 \sin(n\pi/2)}{n\pi/2} \) and the computed mean-square error is:

<table>
<thead>
<tr>
<th>N</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>\varepsilon^2</td>
<td>0.189</td>
<td>0.099</td>
<td>0.067</td>
<td>0.050</td>
<td>0.040</td>
</tr>
</tbody>
</table>

This is plotted in the graph shown.

P2.10.4 \( F_n = \frac{1}{T} \int_0^{T/2} f(t) e^{-jn\omega_0 t} \, dt - \frac{1}{T} \int_{T/2}^T f(t+T) e^{-jn\omega_0 t} \, dt. \)

Changing variable in the second integration, let \( x = t+T/2, \) so that

\[ F_n = \frac{1}{T} \int_0^{T/2} f(t) e^{-jn\omega_0 t} \, dt - \frac{1}{T} \int_0^{T/2} f(x) e^{-jn\omega_0 (x+T/2)} \, dx, \]

\[ F_n = (1 - e^{jn\pi}) \frac{1}{T} \int_0^{T/2} f(t) e^{-jn\omega_0 t} \, dt. \]

This result is zero for \( n \) even. The same conclusions also hold for the trigonometric Fourier series.

P2.11.1 Using Eqs. (2.49), (2.52) in Eq. (2.69), we have:

\[ \frac{1}{T} \int_0^T f^2(t) \, dt = a_0^2 + 2 \sum_{n=1}^\infty \frac{1}{4} (a_n-jb_n)(a_n+jb_n) = a_0^2 + \frac{1}{2} \sum_{n=1}^\infty (a_n^2+b_n^2). \]

P2.11.2 Using the result of Problem 2.11.1 and noting that

\[ \frac{1}{T} \int_0^T f^2(t) \, dt = 1, \quad a_0 = 0, \quad b_n = 0, \quad \text{and} \quad a_n = \frac{2 \sin(n\pi/2)}{n\pi/2} \]

for the given waveform \( f(t), \) we have:

\[ \frac{1}{2} \sum_{n=1}^\infty \left[ \frac{2 \sin(n\pi/2)}{(n\pi/2)} \right]^2 = 1, \quad \text{or}, \quad \sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{8}. \]
P2.11.3 The average power is: \( A^2 \tau / T \). Using #2 in Table 2.2 on page 35,

\[
|F_n|^2 = \frac{A^2 \tau^2}{T^2} \left( \frac{\sin(n\pi \tau / T)}{(n\pi \tau / T)} \right)^2.
\]

The first zero crossing point is given by: \( n\pi \tau / T \leq \pi \), or \( n \leq T / \tau \).

Letting \( J = T / \tau \), the ratio \( R_J \) of the average power contained within \( \pm J \) to the total average power [taking \( |F_0|^2 + 2|F_n|^2 \) for \( n \leq J \)]

is: \( R_J = \frac{1}{J} + \frac{2}{J} \sum_{n=1}^{J} \left( \frac{\sin(n\pi / J)}{(n\pi / J)} \right)^2. \)

The numerical results, expressed in percent, are listed below.

<table>
<thead>
<tr>
<th>( J )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_J )</td>
<td>90.53</td>
<td>90.33</td>
<td>90.30</td>
<td>90.29</td>
</tr>
</tbody>
</table>

The answer is practically constant with increasing \( J = T / \tau \), as shown, and approaches an asymptotic value of 90.282%.

P2.11.4 Using the ratio determined in Problem 2.11.3, we have:

\[
R_n = \frac{1}{J} + \frac{2}{J} \sum_{n=1}^{J} \left( \frac{\sin(n\pi / J)}{(n\pi / J)} \right)^2, \quad \text{where} \quad J = T / \tau.
\]

Some numerical results are:

\[
\begin{array}{ccccc}
J & 2 & 3 & 4 & 5 \\
\hline
a) 90\% & N \geq & 3 & 4 & 5 & 6 \\
b) 95\% & N \geq & 9 & 11 & 13 & 15 \\
c) 99\% & N \geq & 41 & 51 & 62 & 72 \\
\end{array}
\]

As \( J = T / \tau \) increases, the relative pulse width decreases and it takes more harmonics to achieve a given percentage of the average power.

P2.12.1 a) Let \( v_1(t) = e^{j\omega t} \) and \( v_o(t) = H(\omega) e^{j\omega t} \); substitution into the differential equation gives: \( H(\omega) = (R/L)/(j\omega + R/L) \).

b) Defining a normalized frequency \( x = \omega / (R/L) \), we see that \( |H(\omega)| \) is down by \( 1/\sqrt{2} \) and the phase is lagging by \( \pi / 4 \) at \( x = \pm 1 \).
c) It is of the same form, but with \(1/(RC)\) replacing \((R/L)\).

\[ P2.12.2 \quad \text{Let } f(t) = e^{j\omega t} \text{ and } g(t) = H(\omega) e^{j\omega t}; \text{ then substitution into the given differential equation gives:} \]

\[ [L^2 C(j\omega)^3 + R L C(j\omega)^2 + 2L(j\omega) + R]H(\omega) = R, \]

\[ H(\omega) = \frac{R}{\sqrt{R^2(1 - LC\omega^2) + \omega^2 L^2(2 - LC\omega^2)^2}} \]

\[ |H(\omega)| = \frac{R}{\sqrt{R^2(1 - LC\omega^2) + \omega^2 L^2(2 - LC\omega^2)^2}}; \quad \theta_h(\omega) = -\tan^{-1}\left[ \frac{\omega L(2 - LC\omega^2)}{R(1 - LC\omega^2)} \right] \]

b) \( |H(\omega_1)| = R\sqrt{C/L}, \quad \theta_h(\omega_1) = -\pi/2. \)

c) Letting \(\omega_1 = 1/\sqrt{LC}\), we have: \(H(\omega_1) = -jR\sqrt{C/L}, \quad |H(\omega_1)| = R\sqrt{C/L}, \quad \theta_h(\omega_1) = -\pi/2. \)

\[ P2.13.1 \quad \text{a)} \ G_n = F_n H(n\omega_0) \text{ and} \]

\[ H(n\omega_0) = \begin{cases} \frac{1}{1 + jn} & \text{for } |n| < 1 \\ 0 & \text{elsewhere} \end{cases} \]

so use of Eq. (2.77) and combining exponential terms gives:

\[ g(t) = (1 - e^{-a}) + \frac{\sqrt{2} a^2}{a^2 + \pi^2} \cos(\pi t - \pi/2). \]

b) A one-ohm resistance is assumed, so that use of Eq. (2.78) gives:

\[ P_g = (1 - e^{-a})^2 + \left[ a^2(1 + e^{-a})/(a^2 + \pi^2) \right]^2. \]

\[ P2.13.2 \quad H(\omega) = (1 + j\omega RC)^{-1}; \quad H(n\omega_0) = (1 + j4\pi n RC)^{-1}. \]

a) \( H(\omega_0) = \frac{1}{1 + j/4}; \quad g(t) = \frac{10}{\sqrt{17}} \cos(4\pi t - 76^\circ). \)

b) \( H(\omega_0) = \frac{1}{1 + j}; \quad g(t) = \frac{10}{\sqrt{2}} \cos(4\pi t - 45^\circ). \)

c) \( H(\omega_0) = \frac{1}{1 + j/4}; \quad g(t) = \frac{40}{\sqrt{17}} \cos(4\pi t - 14^\circ). \)

\[ P2.14.1 \quad \text{a)} \ a_0 = (aA T/2)/T = aA/2, \quad b_n = 0 \text{ by inspection (even symmetry),} \]

\[ a_n = \frac{4}{T} \int_0^{aT/4} A \cos n\omega_0 t \, dt = aA \frac{\sin(n\pi/2)}{(n\pi/2)}. \]

b) \( a_2 = \frac{A}{\pi} \sin \pi(1 - \epsilon) = \frac{A}{\pi} \sin \pi \epsilon \approx A \epsilon \quad \text{for } \epsilon \ll 1. \)

c) \( a_3 = \frac{2A}{3\pi} \sin[(1 - \epsilon)3\pi/2] = -\frac{2A}{3\pi} \cos(\epsilon3\pi/2) \approx -\frac{2A}{3\pi} \quad \text{for } \epsilon \ll 1. \)
\[ P2.14.2 \quad a) \quad a_0 = 0 \text{ by inspection, } a_n = 0 \text{ (as result of odd symmetry)}, \]

\[ b_n = \frac{4}{T} \int_0^{T/2} A (\sin \omega_0 t + \epsilon) \sin n\omega_0 t \, dt = \begin{cases} A + 4A\epsilon / \pi & n=1 \\ 4A\epsilon / (\pi n) & n>1, \text{ odd} \\ 0 & n \text{ even} \end{cases}. \]

Using Eq. (2.81) and using the series in Problem 2.11.2, we get:

\[ \text{THD} = \frac{(4\epsilon / \pi)^2 \sum_{n=3, \text{odd}}^\infty \frac{1}{n^2}}{(1 + 4\epsilon / \pi)^2} = \frac{(4\epsilon / \pi)^2}{(1 + 4\epsilon / \pi)^2} \left( \frac{\pi^2}{8} - 1 \right). \]

b) For \( \epsilon = 0.10 \), this result gives: THD = 0.298%.

\[ P2.14.3 \quad a) \quad a_0 = 0 \text{ by inspection, } a_n = 0 \text{ (as result of odd symmetry)}, \]

\[ b_n = \frac{4}{T} \int_0^T \sin n\omega_0 t \, dt = \frac{2a}{\pi\omega_0 n^2} (\sin n\omega_0 T - n\omega_0 T \cos n\omega_0 T). \]

b) \[ \frac{\partial b_n}{\partial \tau} = \frac{2a}{\pi\omega_0 n^2} (n\omega_0 \tau \sin n\omega_0 \tau); \text{ we set: } \frac{\partial b_n}{\partial \tau} = 0, \text{ from which: } \tau_o = \frac{T}{2n}. \]

c) Average power is: \( P = \frac{2}{T} \int_0^T (at)^2 \, dt = \frac{2a^2 T^3}{3T} \) and

\[ [b_n]_{\text{opt}} = \frac{2a}{\pi\omega_0 n^2} (\sin \pi - \pi \cos \pi) = \frac{2a}{\omega_0 n^2}, \text{ so desired ratio is: } \]

\[ \frac{b_n^2 / 2}{P} = \frac{1}{2} \left( \frac{2a}{\omega_0 n^2} \right)^2 \left( \frac{3T}{2a^2 T^3} \right) = \frac{6}{\pi^2 n}. \]

\[ P2.15.1 \quad a) \quad T = 2\pi / \pi = 2 \text{ seconds.} \]

b) \( f(0) = e^{-j3\pi(0)} + 2 e^{-j2\pi(0)} + 2 e^{j2\pi(0)} + e^{j3\pi(0)} = 6. \)

c) \( f(1/2) = e^{-j3\pi(1/2)} + 2 e^{-j2\pi(1/2)} + 2 e^{j2\pi(1/2)} + e^{j3\pi(1/2)} = -4. \)

d) \( g_n = \frac{1}{2T} \int_{-T}^T f(t/2) e^{-jn[2\pi/(2T)]t} \, dt = \frac{1}{T} \int_{-T/2}^{T/2} f(x) e^{-jn\omega_0 x} \, dx = F_n. \)

\[ g_n \]

\[ \begin{array}{ccccccc}
-3\pi/2 & -\pi & 0 & \pi & 3\pi/2 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
2 & 1 & & & \\
\end{array} \]

\[ n\omega_0 \]

Thus the line spectrum is the same as that for the \( F_n \), with the exception of a change in the frequency scaling (horizontal axis).

\[ P2.15.2 \quad \text{From Problem 2.7.1 and } a = 1, \quad F_0 = 0.632, \quad F_1 = 0.126, \]

\[ 13 \]
\[ F_2 = 0.016, \quad F_3 = 0.015. \text{ Thus we have: } a_0 = 0.632, \quad a_1 = 0.252, \quad a_2 = 0.031, \quad a_3 = 0.030. \text{ The graphs are shown below.} \]

\[ \begin{align*}
\text{a}_n & \quad \text{F}_n \\
0 & \quad 1 & \quad 2 & \quad 3 & \quad 0 & \quad -3 & \quad -2 & \quad -1 & \quad 0 & \quad 1 & \quad 2 & \quad 3
\end{align*} \]

**P2.15.3**

a) Using #5 in Table 2.2, p. 35, we get: \( a_0 = F_0 = 1/\pi, \)
\[ a_n = 2\Re\{F_n\} = \frac{2}{\pi(1 - n^2)} \]
for \( n \) even, 0 otherwise,
\[ b_n = -2\Im\{F_n\} = 1/2 \]
for \( n = 1, \) 0 otherwise,
so that:
\[ g(t) = \frac{1}{\pi} + \sum_{n=1}^{\infty} \frac{2}{\pi(1 - n^2)} \cos 2\pi nt + \frac{1}{2} \sin 2\pi t \]

b) Graphs of the \( c_n \)'s are shown for both \( f(t) \) and \( g(t). \)

\[ \begin{align*}
\text{c}_1 & \quad \text{c}_2 \\
0 & \quad 2 & \quad 4 & \quad 6 & \quad 0 & \quad 2 & \quad 4 & \quad 6
\end{align*} \]

**P2.15.4**

Based on the first three facts, we can write:
\[ f(t) = a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + a_4 \cos 4\omega_0 t; \]
\[ f(0) = a_2 \cos(0) + a_3 \cos(0) + a_4 \cos(0) = a_2 + a_3 + a_4 = 4, \]
\[ f(T/4) = a_2 \cos(\pi) + a_3 \cos(3\pi/2) + a_4 \cos(2\pi) = -a_2 + a_4 = -1, \]
\[ f(T/2) = a_2 \cos(2\pi) + a_3 \cos(3\pi) + a_4 \cos(4\pi) = a_2 - a_3 + a_4 = 2, \]
from which: \( a_2 = 2, \quad a_3 = 1, \quad a_4 = 1. \) The corresponding line spectra are shown.
b) \[
\frac{(1)^2 + (1)^2}{(2)^2 + (1)^2 + (1)^2} = \frac{1}{3} = 33.3\%.
\]

**P2.16.1, 2, 3** Computer problems.

**P2.17.1, 2** Computer problems.

**P2.18.1** If \( \tau = 0 \) the train of impulses has even symmetry, so that the \( b_n \)'s = 0 and:

\[
a_0 = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) \, dt = \frac{1}{T}; \quad a_n = \frac{2}{T} \int_{-T/2}^{T/2} \delta(t) \cos n\omega_0 t \, dt = \frac{2}{T}.
\]

Using a delay of \( \tau \) units, we obtain the answer given.

**P2.18.2** a) \( e^{-2} \cos[\pi(2-1)] = -0.135 \);  b) \( e^{-3} u(t-3) = 0.05 u(t-3) \);

c) \( (4)^2 + 4 = 20 \); d) \( [(2)^2 + 4]/2 = 4 \); e) \( (3)^3/3 + 4(3) = 21 \).

**P2.18.3** a) \( \int_0^2 t \, dt = 2 \); b) \( \int_0^2 t \, dt = 2 \);

c) \( \int_{t_1}^\infty e^{-(t-t_1)} \, dt = \int_0^\infty e^{-x} \, dx = 1 \); d) \( e^{-(t_0-t_1)} u(t_0-t_1) \); e) \( \int_0^\infty e^{-2t} \, dt = \frac{1}{2} \).

**P2.18.4** a) Assuming continuity of \( f(t) \) and using integration by parts,

\[
\int_{-\infty}^{\infty} f(t) \delta'(t) \, dt = f(t) \delta(t) \bigg|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t) f'(t) \, dt = -f'(0).
\]

b) Assuming continuity of \( f(t) \), using continued integration by parts, and recognizing that \( \delta^{(n)}(t) f(t) = 0 \), we have:

\[
\int_{-\infty}^{\infty} f(t) \delta^n(t) \, dt = f(t) \delta'(t) \bigg|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \delta(t) f'(t) \, dt = f''(0), \text{ etc.}
\]

c) \( \int_a^b t \delta'(t) \, dt = t \delta(t) \bigg|_a^b - \int_a^b \delta(t) \, dt = -\int_a^b \delta(t) \, dt \); therefore \( t \delta'(t) = -\delta(t) \) within the operation of integration.

**P2.19.1** a) In terms of the response to two step functions, we get:

\[
g(t) = \frac{1}{\Delta t} \left[ (1 - e^{-t/RC}) u(t) - (1 - e^{\Delta t/RC} e^{-t/RC}) u(t-\Delta t) \right].
\]

b) Using the fact that \( \exp(\Delta t/RC) \approx 1 + \Delta t/RC \) for small \( \Delta t/RC \),
\[ h(t) = \lim_{\Delta \tau \to 0} \left( 1 - e^{-t/RC} \right) \left[ u(t) - u(t-\Delta \tau) \right] + \frac{1}{RC} e^{-t/RC} u(t-\Delta \tau), \]

\[ h(t) = (1 - e^{-t/RC}) \delta(t) + \frac{1}{RC} e^{-t/RC} u(t) = \frac{1}{RC} e^{-t/RC} u(t). \]

c) \( H(\omega) \) describes a voltage divider, so it is dimensionless. From the results of part b) above, the dimensions of \( h(t) \) are in \((\text{sec.})^{-1}\).

P2.19.2  

a) \( g(t) = \int_{-\infty}^{\infty} h(t) \, dt = \int_{0}^{t} a \, e^{-at} \, dt = (1 - e^{-at})u(t). \)

b) \( g(t) = \frac{1}{2} \int_{-\infty}^{\infty} h(\tau) \, e^{j\omega_{0}(t-\tau)} \, d\tau + \frac{1}{2} \int_{-\infty}^{\infty} h(\tau) \, e^{-j\omega_{0}(t+\tau)} \, d\tau. \)

Assuming that \( h(t) \) is real-valued, we can rewrite this expression as:

\[ g(t) = \frac{1}{2} \left| \int_{-\infty}^{\infty} h(\tau) \, e^{-j\omega_{0} \tau} \, d\tau \right| \quad \text{and} \quad \frac{1}{2} \left[ \int_{-\infty}^{\infty} h(\tau) \, e^{-j\omega_{0} \tau} \, d\tau \right]^*, \]

\[ g(t) = \frac{1}{2} \left| \int_{-\infty}^{\infty} h(\tau) \, e^{-j\omega_{0} \tau} \, d\tau \right| e^{j\theta} + \frac{1}{2} \int_{-\infty}^{\infty} h(\tau) \, e^{-j\omega_{0} \tau} \, d\tau \mid e^{-j\theta}, \]

\[ g(t) = \int_{-\infty}^{\infty} h(\tau) \, e^{-j\omega_{0} \tau} \, d\tau \mid \cos(\omega_{0} t + \theta). \]

In Chapter 3, we identify the above integral as \( H(\omega_{0}) \), verifying the methods used in Section 2.13.

P2.20.1

<table>
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<tr>
<th>Order of derivative in which impulses first occur</th>
<th>Rate of convergence (for large ( n ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 1</td>
<td>( 1/n )</td>
</tr>
<tr>
<td>b) 1</td>
<td>( 1/n )</td>
</tr>
<tr>
<td>c) 2</td>
<td>( 1/n^2 )</td>
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P2.20.2

<table>
<thead>
<tr>
<th>Order of derivative in which impulses first occur</th>
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</tr>
<tr>
<td>c) 1</td>
<td>( 1/n )</td>
</tr>
</tbody>
</table>

[Note, however, that it is \( 1/n^2 \) for the waveform in Problem 2.15.3.]